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On the use of rotational splitting asymmetries to probe the internal rotation profile of stars. Application to β Cephei stars

J.C. Suárez[1,⋆], **L. Andrade**[2], **M.J. Goupil**[3], and **E. Janot-Pacheco**[2]

[1] Instituto de Astrofísica de Andalucía (CSIC), Rotonda de la Astronomía S/N, Granada, 3004, Granada, Spain.
[2] Instituo de Astronomia, Geofísica e Ciências Atmosféricas da Universidade de São Paulo (IAG-USP), Rua do Matão, 1226, São Paulo, Brazil
[3] LESIA, Observatoire de Paris-Meudon, UMR8109, Meudon,France.

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Rotationally-split modes can provide valuable information about the internal rotation profile of stars. This has been used for years to infer the internal rotation behavior of the Sun. The present work discusses the potential additional information that rotationally splitting asymmetries may provide when studying the internal rotation profile of stars. We present here some preliminary results of a method, currently under development, which intends: 1) to understand the variation of the rotational splitting asymmetries in terms of physical processes acting on the angular momentum distribution in the stellar interior, and 2) how this information can be used to better constrain the internal rotation profile of the stars. The accomplishment of these two objectives should allow us to better use asteroseismology as a test-bench of the different theories describing the angular momentum distribution and evolution in the stellar interiors.

1 Introduction

The study of internal rotation of stars is one of the main issues in stellar physics. Rotation is present in almost all the stars, and it interacts with other physical processes acting in the stellar interior. In particular, understand the transport of angular momentum in the interior of stars is crucial to correctly and precisely describe the evolution. Turbulence, meridional circulation, mixing of elements responsible of different μ gradients during evolution, dynamo effects due to the presence of magnetic fields, etc., are some of the physical phenomena and processes affected by rotation (see e.g. Goupil et al. 2005, Goupil 2009, and Goupil & Talon 2009, for a review on this topic).

Nowadays, it is possible to probe the internal structure of stars thanks to asteroseismology. In what regards rotation, progress has been made, during the last decades, on the knowledge about the rotation-pulsation interaction. Up to now, this problem has been tackled using the perturbation techniques to compute the stellar oscillations. These methods (see e.g. Dziembowski & Goode 1992; Soufi et al. 1998; Suárez, Goupil & Morel 2006) are only valid for slow-to-moderate rotators (see a review on the effects of rotation on stellar p modes by Goupil 2009). For faster rotators, the distortion of the stellar structure due to the centrifugal is too large and invalids the perturbation techniques. Non-perturbative approaches must thus be considered (Lignières et al. 2006; Reese et al. 2006).

The present work consider slow-to-moderate rotators, i.e. those for which the parameters $\epsilon = \Omega/(G\,M/R^3)^{1/2}$ and $\mu = \Omega/\nu_{n,\ell}$ are small, i.e. the stellar structure is not significantly deformed by the centrifugal force, and oscillation frequencies are much larger than the angular rotation rate, respectively. The failure of the perturbative approach comes first for high radial-order frequency modes. We restrict thus this study to rotating stars showing oscillations in a relatively low-order frequency domain (low g- and p modes, or mixed modes), like some δ Scuti stars, γ Doradus stars, solar-like stars and some massive stars like β Cephei.

The heterogeneity of internal structures (changing with evolution and different from one star to another) and processes therein, are strong arguments to assume non-uniform rotation. It is thus necessary to take into account possible variations (in the radial or angular directions) of the angular momentum transport, and thus in the shape of the internal rotation profile. To do so, we use the oscillation code FILOU (Suárez 2002; Suárez & Goupil 2008) which corrects the oscillation frequencies for up to second-order effects of rotation (including near degeneracy effects) in presence of radial differential rotation. The study of radial differential rotation has also been used for the asteroseismic studies, e.g. Casas (2006, 2009), Fox Machado (2006), Bruntt et al. (2007a, 2007b); Suárez et al. (2005a), Suárez et al. (2006b); Suárez et al. (2007), for δ Scuti stars, Rodríguez et al. (2006a, 2006b), Uytterhoeven et al. (2008), Moya et al. (2005), Suárez et al. (2005b), for γ Doradus stars, or Suárez et al. (2009) for β Cephei stars.

⋆ Corresponding author: e-mail: jcsuarez@iaa.es

This technique is applied in the present work to analyze the asymmetries of the mode splittings due to rotation. In particular, we are interested in examining the behavior of the rotational splitting and their asymmetries for different mode types (g and p), in presence of radial differential rotation, i.e. to understand physically how variations of the internal rotation profile affect the splitting asymmetries.

2 Rotational splittings

Rotational splittings are being used so far, using inversion techniques, to determine the internal rotation of stars. The most succeeded example of this is the Sun, for which, the internal rotation profile is known precisely thanks to helioseismic inversions (Thompson et al. 1996).

In the framework of the perturbation theory, rotational splittings correspond to the first-order in Ω (rotational frequency) correction to the oscillation frequency $\omega_{n,\ell,m}$. In that case, the Coriolis acceleration ($2\Omega\times\mathbf{v}$) dominates, and the frequency correction term can be written as

$$\omega_{1,m} = \int_0^R \int_0^\pi K_m(r,\theta)\,\Omega(r,\theta)\mathrm{d}\theta\mathrm{d}r \tag{1}$$

where $K_m(r,\theta)$ is the rotational splitting kernel, and R the stellar radius. We use this expression to define the generalized splitting S_m as

$$S_m = \frac{\omega_{1,m}-\omega_{1,-m}}{2m} \tag{2}$$

For the sake of simplicity, we assume here shellular rotation, i.e. $\Omega(r) = \Omega_\mathrm{s}[1+\eta_0(r)]$, so that the splitting becomes independent of m, and its kernel can be written as

$$K(r) = \frac{\xi_r^2 - 2\xi_r\xi_h + (\Lambda-1)\xi_h^2}{\xi_r^2+\Lambda\xi_h^2} r^2\rho_0 \tag{3}$$

where ξ_r and ξ_h correspond to the radial and horizontal components of the eigenfunction, respectively. The density of the star (unperturbed) is represented by ρ_0, the radial distance by r, and $\Lambda=\ell(\ell+1)$. The denominator of this equation is generally known as the mode inertia. When rotation is assumed uniform, the rotational splitting simplifies to

$$S = \Omega\int_0^R K(r)\mathrm{d}r \tag{4}$$

For a shellular rotation, the rotational splitting can be written in the form given by Suárez et al. (2006a), as

$$S = m\,\Omega_\mathrm{s}\,(C_\mathrm{L} - 1 - J_0) \tag{5}$$

where C_L is the Ledoux constant, and J_0 is a η dependent integral (which is null for uniform rotation), defined by

$$J_0 = \frac{1}{I_0}\int_0^R \eta_0(r)[y_{01}^2 + \Lambda\, z_{01}^2 - 2y_{01}z_{01} - z_{01}^2]\,\rho_0\, r^4\,\mathrm{d}r\,. \tag{6}$$

with I_0 being the mode inertia previously defined in Eq. 3.

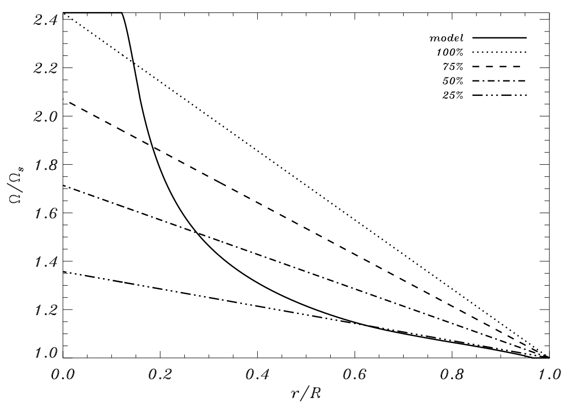

Fig. 1 Two types of rotation profile: linear and shellular (local conservation of angular momentum).The linear profiles were constructed keeping the same rotational frequency at the stellar surface, but varying it at the core, approximately at 100%, 75%, 50%, and 25% of the original one $\nu_\Omega = 2.42$. These are compared with a shellular type rotation profile with the same rotational frequency at the surface.

3 Rotational splitting asymmetries

It often observed that rotational splitting are not perfectly symmetric (with respect to axisymmetric modes). Mathematically, asymmetries of splitting can be defined, in its generalized form, as

$$A_m = \omega_{-m} + \omega_{+m} - 2\omega_0\,. \tag{7}$$

Doing some simple algebra, it can be shown that splitting asymmetries are dependent only upon second-order (in Ω) terms, particularly on

$$A_m = m^2\,X_2\,\frac{{\Omega_\mathrm{s}}^2}{\omega_0} \tag{8}$$

where expressions for X_2 (the Saio form) can be found in e.g. Suárez et al. 2006a or Goupil (2009) and references therein. Then, it thus possible to construct the kernel for the splitting asymmetries, K_2 such as

$$A_m = m^2\int_0^R \Omega^2(r)K_2(r)r^2\rho_0\mathrm{d}r \tag{9}$$

which is implicitly dependent on the rotation profile η_0. In this work we are attempting to examine K_2 in detail. To do so, it is necessary to investigate the behavior of A_m for different mode types and its sensitivity to variations of the rotation profile in different zones of the stellar interior.

4 Probing rotation profiles using splitting asymmetries

The study of the splitting asymmetry kernel is not an easy task. Structure and pulsation variables, together with their derivatives, make the K_2 mathematical expression complex,

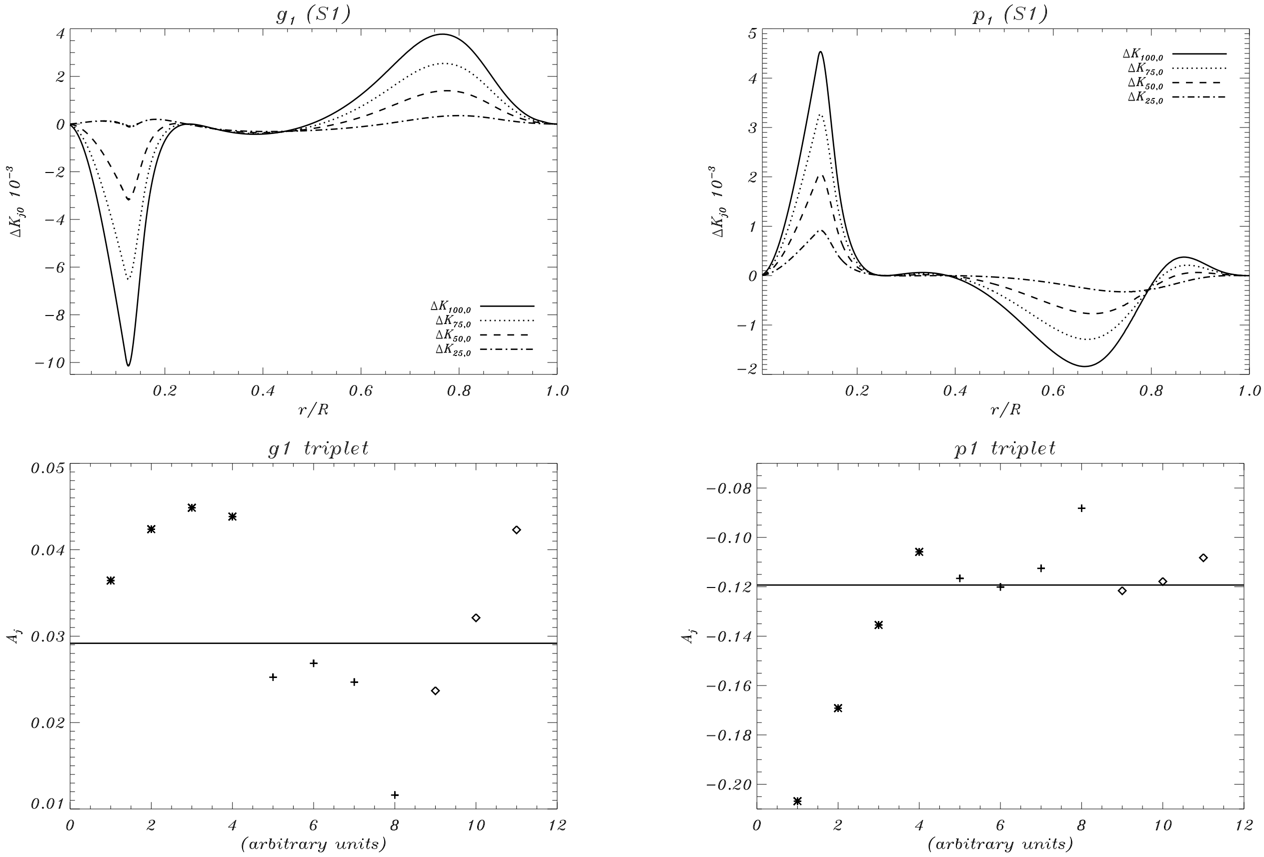

Fig. 2 Absolute differences ΔK (top panels) between the kernels calculated for linear rotation profiles and the reference one (shellular profile), for the modes g1 (left), p1 (right). The K subindices correspond the percentage of the core rotation rate of the reference model (see Fig. 1) Bottom panels show the theoretical asymmetries found for the different modes and rotation profiles. The horizontal line represents the splitting asymmetry of the g1 and p1 modes (left and right panels, respectively) found for the reference model. In order to identify the models we consider the following representation: asterisks for linear profiles, crosses for smoothed shellular rotation profiles, and rhombus for shellular rotation profiles.

and hamper the interpretation of its variations in terms of physics. It is thus rather difficult to invert the rotation profile by using the complete expression of K_2 (see appendix of Suárez et al. 2006a). On the other hand, one expects that the *pulsation terms* in the asymmetry kernel are different for different mode types (pure p and g modes, and mixed modes). It is thus worth studying the impact of variations in the rotation profile on the predicted asymmetries for the different mode types. For the sake of simplicity, we will consider models typically representative of β Cephei stars.

To do so, we constructed $8\,\mathrm{M}_\odot$ equilibrium models with the evolutionary code CESAM (Morel 1997) for two different rotation profiles: 1) shellular rotation (we used this model as the reference), obtained assuming local conservation of the angular momentum during the stellar evolution, and 2) linear profiles, which drastically varies (with respect to reference model) the rotational velocity in all the stellar interior. We forced all the models to have the same rotational frequency at the surface (see Fig. 1). In this latter case, we computed models with rotational frequency of the core $\nu = \Omega_c/\Omega_s = \nu_{\mathrm{ref}}, 0.75\,\nu_{\mathrm{ref}}, 0.50\,\nu_{\mathrm{ref}}$, and $0.25\,\nu_{\mathrm{ref}}$, respectively.

We first analyzed the impact of the variations in the rotation profile on the splitting. We used the oscillation code FILOU (Suárez 2002, Suárez et al. 2008) to compute the oscillations. Figure 2 (top panels) shows the kernel differences between the linear rotation profiles models with the reference one $\Delta K_{j0} = K_j(r) - K_{\mathrm{ref}}(r)$ predicted for g1 and p1 split modes. As expected, for both type of modes, the largest differences are located around the main peaks of the corresponding kernels (near the stellar core for g1, and towards the stellar surface for p1). Even if p1 is a mixed mode (with energy in both the surface and the core), ΔK_{j0} are larger for g1. This would help to disentangle (and thereby identify) low frequency g and p modes, which are typically observed in β Cephei stars.

We then calculated, for each triplet, the corresponding splitting asymmetry according to Eq. 7 and compared them with the reference one (Fig. 2, bottom panels). For illustration, we included in the figure the results for models computed with *modified* shellular-like rotation profiles (simi-

lar to the reference model but modifying the rotational frequency of the core), and smoothed shellular-like rotation profiles, used to discard numerical problems with derivatives near the convective core edge.

A first glance indicates that the largest relative variations from the reference asymmetry corresponds to the g1 triplet (up to 60%), and remains around (up to 40%) for the p1 triplet. As expected, the largest departs from the reference asymmetry, A_0 corresponds to the linear rotation profile models (asterisks) which overestimate it. In the case of smoothed shellular-like models, the results are very close to the reference model for both modes with some marginal exception. Finally, the asymmetry found for the non-smoothed profiles (rhombus) varies significantly for g1, but remain very close to the reference asymmetry value (p1 triplet).

Some preliminary conclusions can be extracted: rotational splitting asymmetries are strongly sensitive to changes of the rotation profile, and such a sensitivity is, somehow, differential, depending on the type of mode (g, p or mixed mode). Physically, the asymmetries are dependent of second-order terms in the oscillation frequency that mainly account for the distortion caused by the centrifugal force. However, we still need to understand how the structure and pulsation variables are related to the asymmetry and its variations that allow us to construct a simplified kernel for the splitting asymmetry (work in progress). Moreover, thanks to the very precise data from space missions like *CoRoT* (Baglin et al. 2003), *Kepler* (Gilliland et al. 2010), it is possible to measure with unprecedent precision the rotational splittings and their asymmetries. This, together with the increase of the number of detected modes (see e.g. García Hernández et al. 2009, Poretti et al. 2009), will definitely help for the present study.

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